The book deals with an area that is not the subject of widespread research interest. At no point did the book surprise me, nor did it offer new perspectives. Being conceived as lecture notes (with exercises) it was probably not intended to have these effects. It could be of interest to students and researchers in approximation theory, systems theory, or numerical analysis (all the methods are illustrated with numerical examples). Certainly not all the possibilities have been explored and the methods proposed may be considered as samples of the potential applicability of orthogonal function systems in this area. It is unfortunate, however, that more exciting modern orthogonal systems such as wavelets are not even mentioned.

Adhemar Bultheel
E-mail: Adhemar.Bultheel@cs.kuleuven.ac.be

T. Ransford, *Potential Theory in the Complex Plane*, London Mathematical Society Student Texts **28**, Cambridge Univ. Press, Cambridge, UK, 1995, x + 232 pp.

This is a marvelously written text containing the fundamentals of logarithmic potentials. Although there are many monographs on potential theory, very often the two-dimensional theory is treated (if treated at all) somewhat left-handedly as the limit case of more general kernels, while the theorems are mentioned in passing as exceptions to the general, in most cases rather abstract, theory. This should not be so, for the two-dimensional theory was the very model for the more abstract developments and it has an extremely strong relationship with complex analysis, as is beautifully illustrated in this book. This is the first of its kind in the sense that it concentrates on the two-dimensional theory, allowing students to discover the beauty and utility of the theory without too much background or too many abstractions. From the preface: “Indeed it was consciously written as the book that I should have liked to read all those years ago.”

The book starts out with the basics of harmonic functions: mean value property, maximum principle, Poisson integral, and Harnack’s inequality. This chapter culminates in the recent proof of Picard’s theorem by J. Lewis. The next chapter is on subharmonic functions and the maximum principle, the latter of which is used to prove several forms of the Phragmén–Lindelöf principle.

Chapter 3 deals with logarithmic potentials, extremal measures, and polar sets. Here the natural applications are in connection with removable singularities. In fact, this is the first example that shows the relevance of potential theory to function theory. Polar sets are the exceptional sets of potential theory much like sets of zero measure are the exceptional sets in measure theory. Polar sets are where logarithmic potentials can take the value infinity, and yet they pop up also as sets of removable singularities: if a function on a region G is locally bounded and holomorphic outside a polar set, then it can be uniquely extended to a holomorphic function on G (this is only the simplest form of the principle).

Chapter 4 deals with the Dirichlet problem. The geometric structure of the boundary of the domain plays an important role, so the so-called regularity property of boundary points forms a central part in the theory. In this chapter the analytic tools, such as harmonic measures, Green functions, and the Poisson–Jensen formula, are already in full swing. The main applications in this chapter are Lindelöf’s theorem on asymptotic values of holomorphic functions (if f is bounded and holomorphic on the upper half-plane and it has a limit value Z along a curve tending to infinity, then $f(z) \to Z$ uniformly as $z \to \infty$ in such a way that $\arg(z)$ lies between two constants $\varepsilon > 0$ and $\pi - \varepsilon$), the Riemann mapping theorem, and continuity properties of conformal mappings at boundary points.

Chapter 5 is devoted to properties of logarithmic capacity, transfinite diameter, and Wiener’s criteria for continuity of logarithmic potentials. Finally, Chapter 6 contains applications in five different directions. The first one is the Riesz–Thorin interpolation theorem on the norm of operators in intermediate spaces. Here subharmonicity appears in the proof, but the flavor of convexity, which is so characteristic of subharmonicity, appears already in the formulation of the theorem: if T is a linear operator with norms M_0 and M_1 between pairs of spaces (L^{p_0}, L^{q_0}) and (L^{p_1}, L^{q_1}), then for any pair (L^p, L^q) with

$$\frac{1}{p}=\frac{1-\theta}{p_0}+\frac{\theta}{p_1}, \qquad \frac{1}{q}=\frac{1-\theta}{q_0}+\frac{\theta}{q_1}; \qquad 0<\theta<1,$$

the norm of T is at most $M_0^{1-\theta}M_1^{\theta}$, i.e.,

$$\|Tf\|_{L^q} \leqslant M_0^{1-\theta}M_1^{\theta}\, \|f\|_{L^p}.$$

The second application is concerning homogeneous polynomials: it is shown that if $p(z_1, ..., z_n)$ is a homogeneous polynomial of n complex variables, and $E_1, ..., E_n$ are sets with fixed two-dimensional Lebesgue measure $m_1, ..., m_n$, then the minimum of

$$\max\{|p(z_1, ..., z_n)| : z_1 \in E_1, ..., z_n \in E_n\}$$

is attained when all the sets E_j are disks around the origin. The next section contains the important result of Keldysh on the approximability of continuous functions on boundaries of a compact set K by real parts of rational functions (more precisely by functions of the form $\operatorname{Re}\, r(z) + a \log |q(z)|$, where a is a real number and r and q are rational functions with poles resp. poles and zeros from a prescribed set which contains at least one point from every connected component of $\overline{\mathbb{C}} \setminus K$). The next section discusses topics concerning subharmonicity in Banach algebras, while the final section concers complex dynamics such as the capacity and equilibrium measure of Julia sets or Green functions and harmonic measures for attractive basins.

Each chapter contains historical notes, and at the end of the sections there are exercises that direct the reader to further properties or applications. A brief appendix discusses that part of measure theory that is needed to read the text conveniently.

I have thoroughly enjoyed reading this small treatise, and I recommend it for all students and scholars interested in analysis.

Vilmos Totik
E-mail: totik@inf.u-szeged.hu or totik@math.usf.edu

A. I. Stepanets, *Classification and Approximation of Periodic Functions*, Mathematics and Its Applications **333**, Kluwer, Dordrecht, 1995, x + 360 pp.

This book deals with the approximation of periodic functions of one variable by Fourier series. The first chapter opens with a thorough discussion of modulus of continuity in both the uniform and L^p-metrics, and then defines various classes of functions that will be used throughout the book. The author argues that we would like to be able to use all the information we have about the properties of a function in order to understand how well it can be approximated by various means. To do this, we need approximation theorems for quite general classes of functions. So, starting with the well-known classes of functions, the author sets about defining, in a systematic manner, certain classes of functions which become more and more general.

The classic work by Timan [2] (now reprinted by Dover for the benefit of future generations of workers in approximation theory) concludes with a discussion of linear processes of approximation theory. This is the starting point of chapter 2 of the present work by